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Some questions remain, for which we have as yet no answer to suggest: the position of the bright-line helium stars and the Wolf-Rayet stars in the line of evolution, the relation between planetary nebulæ and the novæ, whose spectra have points of similarity. More fundamental perhaps is the question of the meaning of the hazy lines in some spectra of Classes A and F, the differences, not of the intensity of the lines but in their definition, which exist among the spectra of these classes, and which were recognized in one of the earliest Harvard classifications. In the light of the advance already made, however, and of the increase in the number and power of our instruments, we may be confident as to the final solution of this problem of the evolution of the stars.

May, 1913.

THE NUMBER OF STARS IN THE UNIVERSE.

By R. H. Tucker.

Many attempts have been made to form an estimate of the number of stars visible in our most powerful telescopes, or to be distinguished with the aid of photography; and speculation has been rife as to whether the universe may extend beyond the limits at which stars have been actually found to exist. It has been impossible to make counts of more than very limited areas, from which, by comparison and approximation, the estimates may be extended to larger areas, and to the whole sky.

Herschel's early set of "gauges" were counts of stars, made in various parts of the sky, with the object of forming a comparative estimate of the "richness" in many directions; thus illustrating, possibly, the form and extent of the universe of visible stars. One of his recorded estimates is five and a half million stars to the fourteenth magnitude, in the sky. But **Struve**, at about the same epoch, estimated twenty million of the same grade.

The most common estimate has been that fifty million would be visible in our best modern telescopes. In a recent number of *Harper's Magazine,* WILLIAMS has given the number as one hundred million, with the aid of photography. It is well known that the photographic plate, with long exposure, will record stars much fainter than are visible with the same telescope. Photography has probably given impressions of stars that would be ranked as twentieth magnitude, on the adopted scale. The greater part of the recently discovered asteroids, from twelfth to sixteenth magnitude, have been found by photographing the sky with comparatively small telescopes. And the latest known additions to the planetary system, the faint satellites of *Jupiter* and *Saturn,* were discovered by photography, and are generally not to be detected with direct vision in the most powerful telescopes existing.

To illustrate the limits of vision of our telescopes, we may start with the sixth magnitude as visible to the naked eye under usual conditions, though a well-trained observer can detect stars one grade fainter, under the best conditions. With a telescope of one inch objective, the ninth magnitude may be seen; a 10-inch objective should show the fourteenth, a 16-inch should show the fifteenth, and our most powerful refractors, with nearly 40-inch objectives, are required to make visible the stars of seventeenth magnitude. The recently equipped five-foot reflector at Mount Wilson should reach nearly to the eighteenth magnitude, and the projected 100-inch reflector, for the same observatory, should extend the visible limit close to the nineteenth magnitude.

The limits of the several parts of the *Durchmusterung* have been noted in previous papers in these *Publications.*[1] For ARGELANDER, ninth magnitude complete, with stars certainly of tenth, but apparently going down to eleventh magnitude on the photometric scale, the faintest that could have been seen with his telescope. SCHÖNFELD and Córdoba could have reached the twelfth magnitude, and the photometric estimates rank some of the stars catalogued as of that grade; the former was planned to be complete to nine and a half, the latter to tenth magnitude. The measured images of the *Cape Photographic Durchmusterung* give limits for different regions, ranging from 9.6 to 11 magnitude, and it is probably complete, nearly, to the ninth magnitude. Its scale, founded upon comparison with the magnitudes of GOULD's zones, does not differ widely from the visual *Durchmusterung* scale.

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[1] *Publications A. S. P.,* No. 147 and No. 148.

This entire project for taking a census of the sky, as it might be described, finally includes in round numbers 315,000 stars for the northern portion, an average of fifteen to the square degree, and about 775,000 stars in the southern sky, an average of thirty-seven to the square degree. The proportion of those enumerated at Córdoba, rising in parts to above sixty to the square degree, if continued throughout the sky, would give two and a half million stars. These averages of distribution include the "Milky Way," in which the density of stars of the fainter grades is twice as great as the average density of the whole sky.

It is often asked, How many stars are known, so as to be recognized by astronomers. Many of the old observers were so familiar with the constellations that all the easily visible naked-eye stars could be located, and in a sense could be named, within their respective families. Some modern observers, comet seekers especially, have familiarized themselves with the constellations in this way, for the part of the sky visible above the horizon at one place. Taken singly, only a few very bright stars or distinctive colored or double stars would have been recognized. But we have this census of the stars to the ninth magnitude, and to the tenth for a part of the sky, and it could be said that the astronomer with a telescope could identify any one of these million stars—it is a common detail of his practice to make such an identification.

The *Durchmusterung* includes all of the stars, practically, that can be precisely observed for direct measures of position, with modern meridian instruments. For the positions of the fainter stars of this scheme, as well as for positions of stars too faint to have been included, recource must be had to differential measurement with larger equatorial telescopes, or to the measurement differentially of photographic plates covering the region.

On the visual *Durchmusterung* has been based the international scheme of meridian-circle observations, in the *Gesellschaft Zones*. These are now complete for the northern sky to 80°, and have been published to 18° south declination, leaving one zone to be completed to 22° south, the limit of SCHÖNFELD'S *Durchmusterung*. Some observations on the same plan, and extending below —22°, were carried on during the administration of the late Dr. J. M. THOME at Córdoba, based upon the Córdoba *Durchmusterung*. The fifteen northern zones contain the meridian-circle places of about 130,000 stars, allowing for the overlapping of the margins of the several zones, 10′ or 20′ in the individual parts. They were planned to be complete to the ninth magnitude, and to contain also as many of the fainter stars as possible that had been previously observed with the meridian circle. The visual scale of ARGELANDER gives an equivalent of about two hundred thousand stars of ninth magnitude in the whole sky, and the tenth magnitude should include about three quarters of a million. The northern Gesellschaft zones would appear to include an equivalent of the enumeration to the magnitude 9.2 of ARGELANDER'S scale, but would not be complete to any limit below 9.0 of the same scale.

The five southern zones, to —22°, allowing for the overlapping margins, and estimating the unpublished zone, should contain about forty thousand stars. They would appear to include an equivalent of the enumeration to 9.1 of ARGELANDER'S scale, which is close to that of SCHÖNFELD at this point. Including GOULD'S *Zone Catalogue,* which gives positions of 73,000 stars of less precision than those of the Gesellschaft scheme, there are accordingly above 240,000 stars, for which fairly good meridian-circle observations are available. The test of modern use of the zone observations shows them to be generally very reliable, and with actual errors not larger than had been anticipated.

For fainter stars, the proposed chart of the whole sky by the international photographic scheme is to extend to the fourteenth magnitude, well within the visual limits of the 13-inch telescopes employed in the execution. The catalogue of star positions, from shorter exposures with the same instruments, is to extend to the eleventh magnitude inclusive. This catalogue has been estimated as likely to contain two million stars for the whole sky, which would be less than the extent of the Córdoba *Durchmusterung,* were that to be carried beyond its limits in the southern sky. For the charts, LOEWY originally estimated thirty million stars to the fourteenth magnitude; but the Greenwich observers, after completing some portions of their section of the scheme, have arrived at an estimate of fourteen million to that grade.

For the estimates of the photometric scale reference may be made to the *Annals of Harvard College Observatory.* On page 178, volume 48, table XXI gives the estimate to the magnitude 13.5, and there would evidently be about nine million stars of the fourteenth magnitude of that scale. On the page following, Professor PICKERING estimates eighteen million to the fifteenth magnitude. The photometric enumeration is smaller than the visual, beginning at the faint telescopic grades. Thus, at 9.0 the summation is about two thirds that of ARGELANDER. The tenth magnitude includes about one half the number which would result from the extension of ARGELANDER'S scale to that grade. There are less than one million in the estimated eleventh magnitude, which is half the estimate of the photographic charts.

There is a very decided drop in the value of the term b, in the formula which has been used to represent the summation of the various grades of stars on the absolute scale of magnitudes as the fainter stars are included. Up to the *Durchmusterung* limits, the value of b has been found to be nearly 4. For the fainter stars, so far estimated, the value of b soon falls to the neighborhood of 2. The value adopted in the photometric summation drops from 3 at eighth magnitude to about 2 at thirteenth magnitude. It appears to me that this decrease in the relative numbers of the fainter stars is an indication not only of the approach to the limit of the power of our telescopes, but may be also an indication of the limit of the actual extent of the stellar universe.

In a recent number of the *Observatory,* January, 1913, there is an estimate by D'ESTERRE, based upon the count of stars on a large number of photographic plates, which he had exposed for records of variable stars, and of possible new stars. He has the count of the stars on his plates to the magnitude 9.2 of ARGELANDER, and the count also to 12.5, the value of *b* in this range of magnitude being 2.6. He concludes with an estimate of nearly ten million stars to the fifteenth magnitude, in the Milky Way. Assuming the distribution in that part to be twice as dense as the average for the whole sky, and that the Milky Way covers a quarter of the sky, this estimate would give twenty million stars to the fifteenth magnitude in the whole sky. For the range of magnitude from 9 to 15, the value of *b* would be 2.15.

From the count of the stars on the charts, which were made here with the 36-inch telescope, for standards of magnitude of the faintest stars visible with that instrument, the proportions of the various grades may be used to estimate the distribution in the sky. But the area covered by the charts is too small to give any considerable weight to the results.[1] The average of 700 stars to the square degree, and the relative proportion of the brighter stars in the charts, would give the following estimates for the whole sky:—

2	million	stars	to	11th	magnitude	$b = 2.59$
6	"	"	"	13th	"	2.08
13	"	"	"	15th	"	1.84
29	"	"	"	17th	"	1.65

The values of *b* here tabulated would result from the summation at ninth magnitude, compared with each successive grade. The estimate, based on this small area, gives too large a number for the stars to ninth magnitude, but at eleventh magnitude the estimate is close to that of the photographic catalogue. For the fainter grades, the estimates appear to be too small.

It is probable that the extent of visibility in our largest telescopes would be about one thousand stars to the square degree, on the average over the whole sky. This is a convenient number to recall, and possibly is as close an estimate as we are justified in making. It would give about forty million stars down to the seventeenth magnitude inclusive. Photographic plates, with large telescopes, might reveal a proportion down to twentieth magnitude that would be equivalent to one hundred million stars, which is again an easy number to fix in mind, if that can be considered as a point in its favor. We have but little basis to estimate the faintest grades, and it will be a long time before our knowledge will be much extended in this direction. Even the count of the stars in one of the great clusters has been found to be exceedingly difficult; by taking the number in small, selected, representative portions, an estimate of the total number may be made. Photographs of the clusters reveal the central part too much blurred by the condensation of the stars to permit of actual counts. We are rather confident that greater telescopic power will not add to the number of stars visible in a very much increased ratio.

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[1] *Publications A. S. P.*, Vol. VIII, page 95.

The proportion of stars to a square degree gives us useful figures in practice, for planning work of research; and the figures from various authorities make a convenient test for the relation of the respective magnitude scales. The proportion varies considerably for different parts of the sky, including some condensed regions outside the Milky Way and some notably bare regions like an area in the constellation *Scorpio,* which used to puzzle us when we first observed the *Durchmusterung* of that region. Below is a tabulation, formed from the various sections of our census and condensed from extensive counts made for this purpose.

For ARGELANDER'S *Northern Durchmusterung,* there is an exact count by tenths, made by LITTROW, and already referred to in a previous note. The article in the *Astrophysical Journal* for May, 1898, gives the basis for the figures of the first ten degrees of Córdoba, to —32°. The scale of Córdoba below —42° had evidently been changed, and a special count of representative portions of the zone —50° has been made, to illustrate the character of the change. Similarly, counts have been made in the zones, —12° of SCHÖNFELD, and —36° of the Cape *Photographic Durchmusterung,* to give the figures of average distribution. For the Harvard Photometry, the figures have been taken from table XXI, volume 48, page 178. Exact counts of SCHÖNFELD and of the Cape are available, for the magnitude 9.0.

NUMBER OF STARS PER SQUARE DEGREE.

Mag.	Arg. N.D.N.	Schön. —12°	Cór. —32°	Cór. —50°	Cape —36°	H. Photom.
7.0	0.39	0.3	0.4	0.6	0.4	0.36
7.5	0.68	0.6	0.8	1.0	0.7	0.64
8.0	1.23	1.0	1.5	1.4	1.5	1.11
8.5	2.34	1.9	3.1	2.4	2.9	1.91
9.0	4.90	4.65	6.7	4.5	5.85	3.26
9.1	5.66	5.5	7.9	5.0	6.8	...
9.2	6.67	6.4	9.4	5.6	8.6	...
9.3	8.19	7.6	10.8	6.3	9.3	...
9.4	9.89	9.0	12.6	7.7	12.0	...
9.5	15.27	11.3	15.6	9.0	12.7	5.46
9.6	...	12.3	19.1	10.6	15.6	...
9.7	...	13.3	23.9	12.8	16.6	...
9.8	...	15.5	29.4	16.6	20.5	...
9.9	...	16.0	34.1	21.7	21.0	...
10.0	...	18.1	55.0	61.7	24.5	9.05
10.5	...	...	...	...	31.4	14.6
11.0	...	...	...	...	...	23.2
11.5	...	...	...	...	...	36
12.0	...	...	...	...	...	54
12.5	...	...	...	...	...	80
13.0	...	...	...	...	...	116
13.5	...	...	...	...	...	164
14	...	...	...	...	...	(225)
15	...	...	...	...	...	(400)
16	...	...	...	...	...	(600)
17	...	...	...	...	...	(800)

The flexible nature of ARGELANDER'S 9.5 is evident in the above table. It is equivalent to the summation to 9¾ of its own scale, and undoubtedly includes many tenth-magnitude stars. This point is often overlooked in the discussion of ARGELANDER'S magnitudes. Thus, in a note by J. J. PLUMMER,[1] when discussing the light of the stars, by including all those catalogued by ARGELANDER to 9.5, he independently arrives at the conclusion that this limiting grade probably extends much below that point.

SCHÖNFELD'S scale appears to be consistent throughout, and his section may really include most of the tenth-magnitude stars. The first two volumes of Córdoba estimate about two tenths brighter than ARGELANDER from eight to nine and a half, and the 9.5 of Córdoba, which agrees in the table with ARGELANDER's enumeration at that point, probably corresponds to stars one quarter of a magnitude fainter, on the visual scale. The ten of Córdoba is equivalent to the summation to 10¼ of its own scale, or probably to the 10½ of the visual scale.

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[1] *Monthly Notices Royal Astronomical Society*, for June, 1877.

The change in the scale of the later volume of Córdoba is evident in the table. There is a lack in the proportion of estimates between ninth and tenth magnitudes, and a great preponderance of the limiting grade, 10, which will also be evident in the casual inspection of the pages of Volume XVIII. This grade contains nearly three times as many stars as were catalogued up to 9.9, and fourteen times as many as are included up to 9.0. The estimates appear to be about three tenths fainter than those of the two earlier volumes, between 9 and 9¾, while the actual limit extends but slightly farther.

The great increase in the number of the fainter grade had been noticed by NEWCOMB, and was commented upon in his note in the *Astronomical Journal,* No. 507, September, 1901. It is not clear, however, that NEWCOMB appreciated the fact that the 10 of Córdoba was also a flexible grade, and was intended to include many stars estimated as fainter than tenth magnitude.

The figures for Cape —36°, which is south of the point where the photographic scheme decidedly enlarged its extension, show fair agreement with the visual estimates. Below tenth magnitude the photographic catalogue contains but few stars of each grade, and the limiting grade evidently varies widely in various parts.

The difference between the photometric and the visual scale, for faint stars, is very evident in the figures of the table. The difference begins to be noticeable at the eighth magnitude. Approximate estimates have been included for the faintest grades visible, although such extension is not to be recommended, in the absence of actual counts sufficient to form a basis for it. The proportion of 800 stars of seventeenth magnitude to a square degree would give, quite precisely, thirty-three million stars in the whole sky, 41,253 square degrees.

One may explain, if it is not possible to reconcile this difference, by the circumstances of the estimates of magnitude.

The visual observer, in endeavoring to grade all the stars in classes, may take too large a difference of brightness to represent the distriction of a quarter of a magnitude, a very small quantity to be estimated, as he goes down in the scale beyond the types of stars from which he must form his magnitude scale. The photometric measures may have developed a tendency in the contrary direction; taking too small a difference of scale reading to represent the difference of a tenth, or of a quarter of a magnitude. The photometer has the great advantage that large differences of brightness can be measured, and used as checks, assuming that the instrument gives consistent and authentic values for the steps down from one type of star to another, at all parts of the scale of brightness.

A great amount of detailed comparison of the photometric and visual D. M. magnitudes has been given in the latest publication of the Harvard College Observatory, Parts 6 and 7 of Volume 72, which were received here after the figures of this note had been prepared.[1]

July 12, 1913.

THE DERIVATION OF THE RADIAL VELOCITY EQUATION.

By G. F. Paddock.

The radial velocity equation representing the component of velocity of binary stars in the direction of the Sun was first derived by Lehmann-Filhes.[2] His method, which follows in brief, was the differentiation of the co-ordinate z, whose origin is at the center of motion and whose direction is along the line of sight. Let—

a = semi-major axis;

e = eccentricity;

i = inclination of the orbit plane with respect to the plane tangent to the celestial sphere;

ω = longitude of periastron from the receding or ascending node;

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[1] In Campbell's "Stellar Motions," recently issued from the Yale University Press, he refers to the number of stars observable in our greatest telescopes as being about one hundred million. (Page 118.)

[2] *Astronomische Nachrichten*, **136**, 17.